Consumer Theory

Preferences
Indifference Curves
Utility Functions

Consumer Theory

<u>*Situation:*</u>
A consumer needs to decide how to spend his income or wealth on different goods with the objective of maximizing his well-being.

For example, assume you earn €800 Euros per month. You pay €300 for rent. How much of the remaining €500 do you spend on books? How much on food that you cook yourself and how much on restaurants? How much on drinks with friends?

Consumer Theory

<u>*General Question:*</u>

- How do consumers <u>*decide*</u> what to buy?
- What determines the (individual, market) demands of goods and services?
- How do the demands of goods and services depend on good prices, income, etc.?

Consumer Theory

In order to describe the consumers problem we need to specify (at least) two things:

<u>*Preferences:*</u> *(today's topic)*
How are alternative consumption bundles ordered? Or in plain language: which goods increase my well-being by a lot and which don't?

<u>*Constraints:*</u> *(next week's topic)*
What is the set of feasible consumption bundles? What can the consumer afford?

Consumer Theory

Preferences: What do I want?

Constraints: What can I afford?

The consumer's preferences and constraints determine her/his optimal choice.

The *consumer's choice* is the *feasible* consumption bundle that *maximizes* the consumer's *welfare / well-being.*

Microeconomics provides us with a formal apparatus that helps us to study these questions and its implications.

Consumer Theory: Preferences

We now introduce a formal language to study consumer choice

A *bundle* is a list of specific quantities of distinct goods and services.

If there are n different goods, we often denote a bundle

$$(x_1, x_2, \ldots, x_n)$$

where x_i is the quantity of good $i \in \{1,2,..,n\}$.

Consumer Theory: Preferences

During the course we will usually study the case of $n = 2$ and use the letters x and y to denote quantities:

$$(x,y)$$

For example, *x* could be number of pizzas and *y* could be glasses of wine.

To find the optimal bundle, the consumer needs to know how much she likes the different goods → preferences

To study preferences systematically, it is useful to first learn how to graphically represent bundles

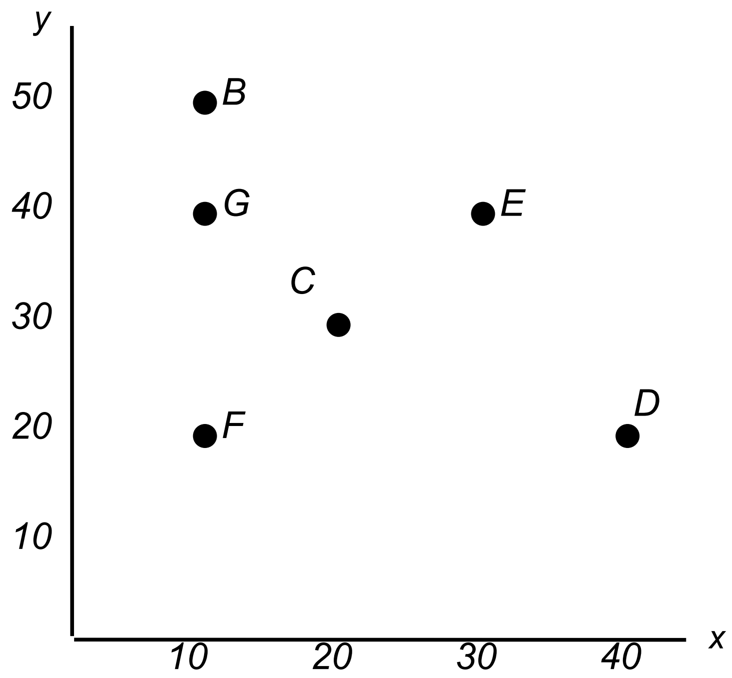
Consumer Theory: Preferences

Take the following example:

Bundle	Units of food (x)	Units of clothes (y)
B	10	50
C	20	30
D	40	20
E	30	40
F	10	20
G	10	40

Consumer Theory: Preferences

For simplicity, we will assume that goods are *perfectly divisible* so that every point in the positive part of the real plane is a possible bundle.



Consumer Theory: Preferences

Let *A = (x,y)* and *B = (x',y')* be two distinct bundles.

$\succsim$ preference relation:
A $\succsim$ B means "A is weakly preferred to B"

$\succ$ strict preference relation:
A $\succ$ B means "A is (strictly) preferred to B"
Formal equivalent: A $\succsim$ B, but not B $\succsim$ A.

~ indifference relation:
A ~ B means "A is indifferent to B"
Formal equivalent: A $\succsim$ B and B $\succsim$ A.

Consumer Theory: Preferences

Examples:

1. *Pareto preferences*:

$$A \gtrsim B \text{ if } x \geq x' \text{ and } y \geq y'$$

2. *Lexicographic preferences* (like a dictionary):

$$A \gtrsim B \text{ if } x > x' \text{ or } [x = x' \text{ and } y \geq y']$$

3. *Goods and "Bads"* (e.g. pollution):

$$A \gtrsim B \text{ if } x - y \geq x' - y'$$

Consumer Theory: Preferences

4. *Perfect substitutes:*

$$A \gtrsim B \text{ if } x + y \geq x' + y'$$

5. *Imperfect substitutes:*

$$A \gtrsim B \text{ if } xy \geq x'y'$$

6. *Perfect Complements:*

$$A \gtrsim B \text{ if } \min\{x,y\} \geq \min\{x',y'\}$$

Consumer Theory: Preferences

Let's put some structure on preference relations

Three basic axioms:

A1: Preferences are *complete* if for all bundles A, B:

$A \succsim B$ or $B \succsim A$ or both.

Meaning:
Consumers can rank any two bundles.

Consumer Theory: Preferences

A2: Preferences are *transitive* if for all bundles *A, B, C*:

$A \succsim B$ *and* $B \succsim C$ *implies* $A \succsim C$

Meaning:
Consumer's preferences do not cycle, that is,

$$A \succ B \succ C \succ A$$

never holds → choice making is consistent!

Why is that important?

Consumer Theory: Preferences

A3: Preferences are *monotone* if
for all bundles A=(x,y) and
B=(x',y'):

$$(x,y) \geq (x',y') \text{ implies } A \succsim B,$$

and

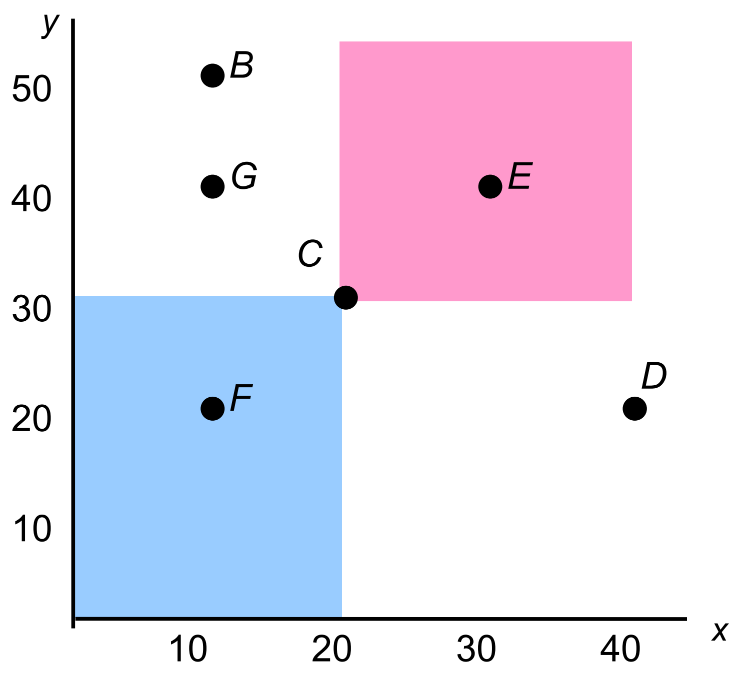
$$(x,y) >> (x',y') \text{ implies } A \succ B.$$

Meaning:

More is better! (the greedy axiom)

Consumer Theory: Preferences

A3 implies that *C* is preferred to F (and to all bundles in the blue area), while E (and all the bundles in the pink area) is preferred to *C and F.*



Consumer Theory: Preferences

Other Axioms:

A4: Preferences are *continuous*:

If A $\gtrsim$ B(n) $\forall$ n and {B(n)} $\rightarrow$ B, then A $\gtrsim$ B.

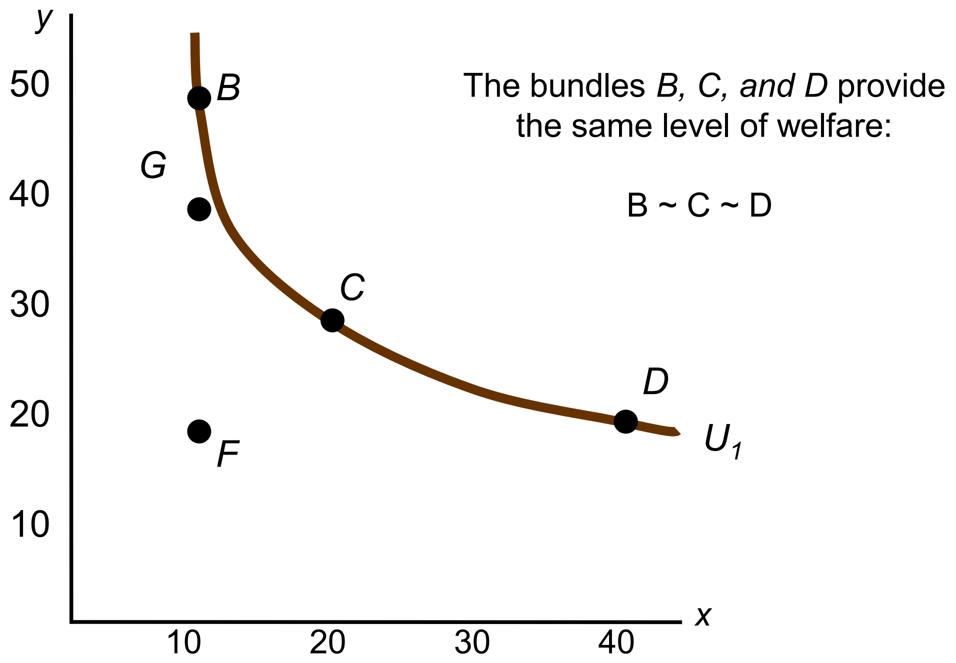
If B(n) $\gtrsim$ A $\forall$ n and {B(n)} $\rightarrow$ B, then B $\gtrsim$ A.

A5: Preferences are *convex:*

If A $\gtrsim$ B and 0 < λ < 1, then [λA + (1- λ)B] $\gtrsim$ B.

Consumer Theory: Indifference Curves

An indifference set or *indifference curve* contains the bundles that provide the same level of satisfaction or welfare for a given individual.



Consumer Theory: Indifference Curves

What kind of structure do the axioms imply?

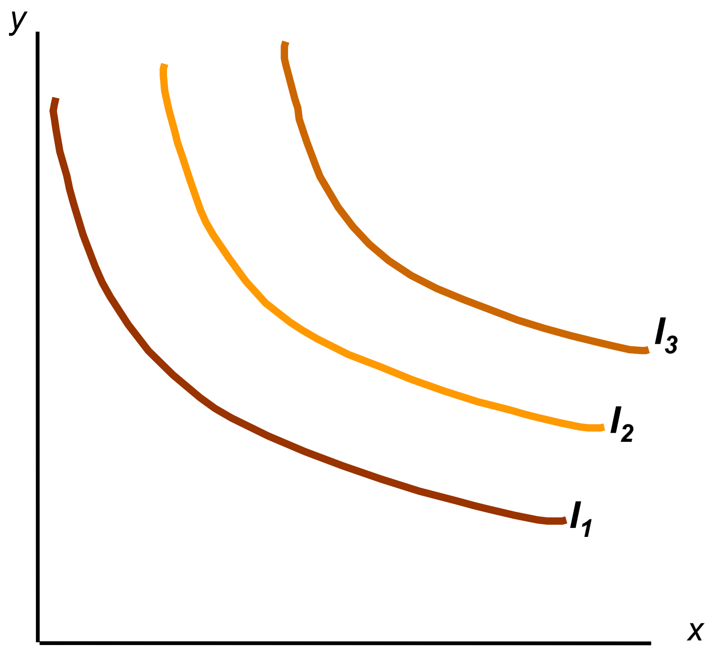
A1: Every bundle is on some indifference curve.

A2: Indifference curves cannot cross.

A3: Indifference curves are decreasing and have no area (that is, *are* curves).

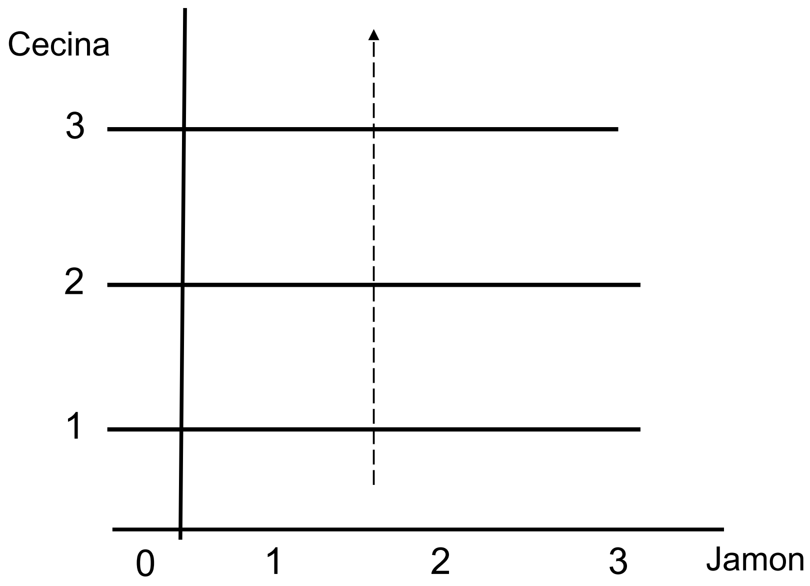
Consumer Theory: Indifference Curves

An *indifference map* provides a description of an individual's preferences by identifying his indifference curves.



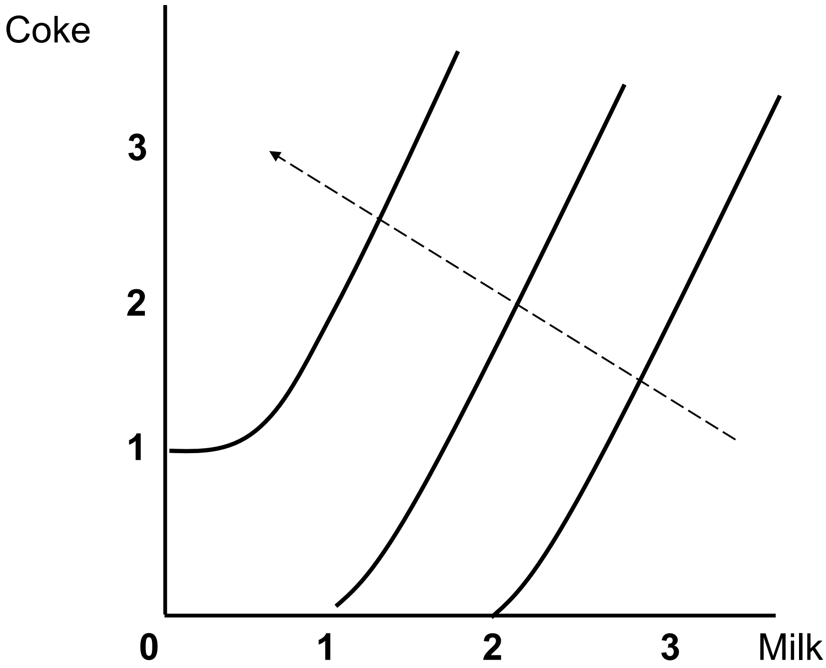
Consumer Theory: Indifference Curves

Indifference map: "I love Cecina, but I just don't care about Jamón." Does this preference satisfy axiom A.3?



Consumer Theory: Indifference Curves

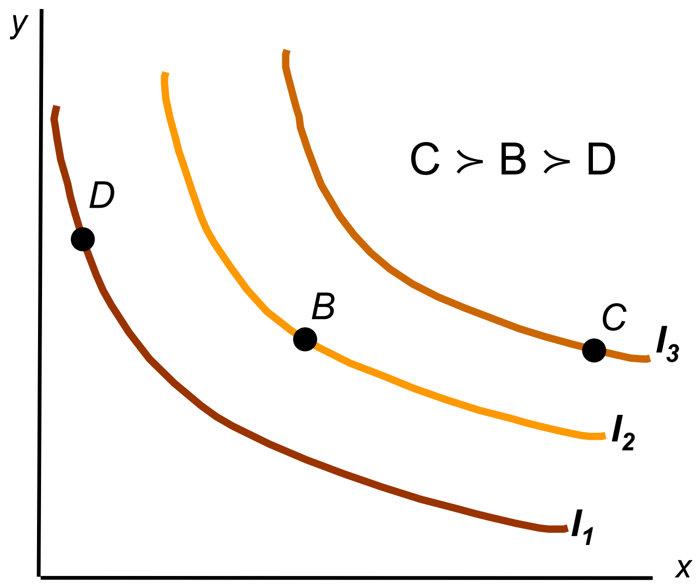
Indifference maps: "I like coke but I really hate milk." Does this preference relation satisfy axiom A.3?



Consumer Theory: Indifference Curves

Other properties of indifference curves implied by the axioms:

A3 implies that indifference curves are *decreasing*: if (x,y) ~ (x',y'), then either x ≤ x' and y ≥ y', or x ≥ x' and y ≤ y'.

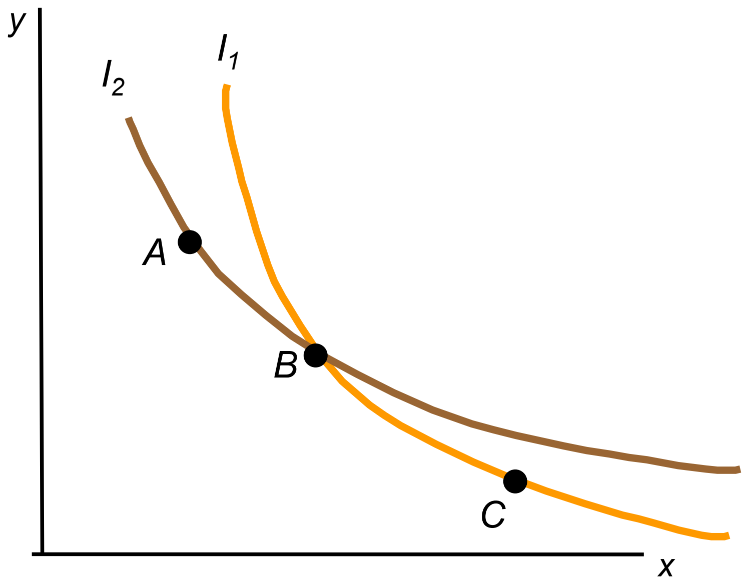


Consumer Theory: Indifference Curves

A2 implies that indifference curves *do not cross*.

If $A \succ C$, then $A \succ C \succsim B$ implies $A \succ B$ (B does not belong to I_2)

If $C \succ A$, then $C \succ A \succsim B$ implies $C \succ B$ (B does not belong to I_1)



Consumer Theory: Utility Functions

The preferences of a consumer $\succcurlyeq$ can be described by an indifference map.

But preference relations are not very handy and with multiple bundles it is generally not really easy to study them.

To have a more tractable model, can we find a function defined on the set of consumption bundles whose map of level curves coincide with the indifference map?

A function u with this property *represents* the consumer preferences and we call it a *utility function*.

Consumer Theory: Utility Functions

The following defines the properties that such a function *u,* that represents some preference relation $\succcurlyeq$, must have:

A function $u: \Re^2_+ \rightarrow \Re$ *represents* the preferences $\succcurlyeq$ of a consumer, if for any two consumption bundles $(x,y),(x',y') \in \Re^2_+$ the following holds:

$$(x,y) \succcurlyeq (x',y') \Leftrightarrow u(x,y) \geq u(x',y')$$

In words:

The function u must lead to the same ranking of bundles as the preference relation $\succcurlyeq$.

Consumer Theory: Utility Functions

What are the advantages of such a utility function?

A utility function provides a *compact representation* of the preferences of a consumer.

However, it does *not* allow us to measure a consumer's *welfare*. The only purpose is to *rank* bundles.

Utility is an *ordinal* concept (i.e., permits to order bundles of goods), but has no other meaning (i.e., larger differences in utility do not indicate larger changes of welfare).

Consumer Theory: Utility Functions

Example:

Assume $u(x,y) = 2x + y$ represents preference relation.

This means that for any two bundles (x,y) and (x',y'), the former is weakly preferred to the latter if

$$2x + y \geq 2x' + y' \Leftrightarrow 2(x - x') + (y - y') \geq 0$$

The function v(x,y) = 3u(x,y) represents the same preferences:

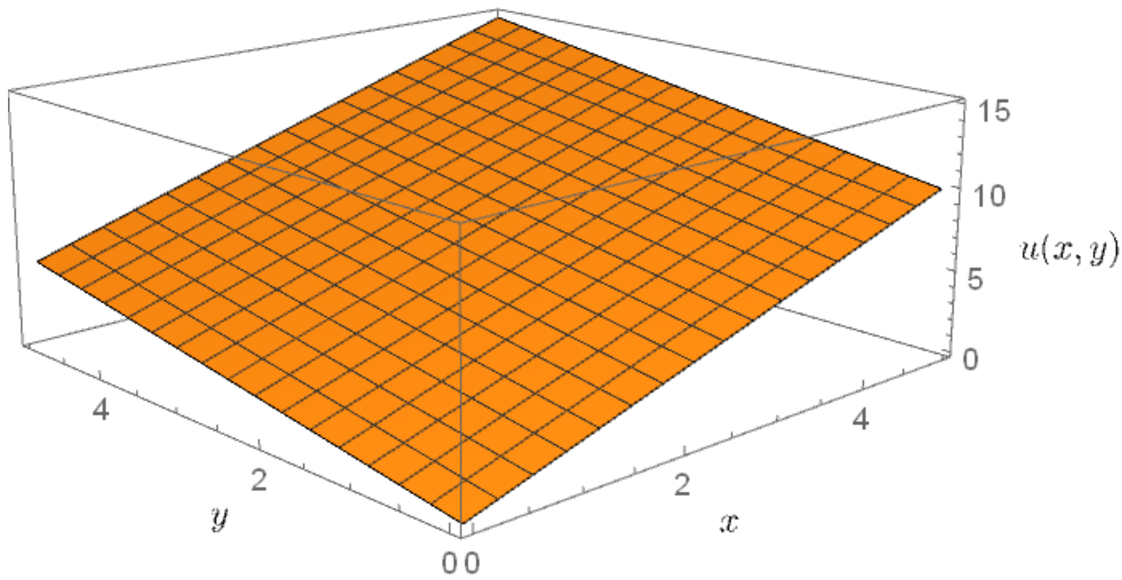
$$3(2x + y) \geq 3(2x' + y') \Leftrightarrow 3[2(x - x') + (y - y')] \geq 0$$

$$\Leftrightarrow 2(x - x') + (y - y') \geq 0$$

But utility and levels differences are larger with v(x,y) than with u(x,y) → the numbers have no intrinsic meaning!

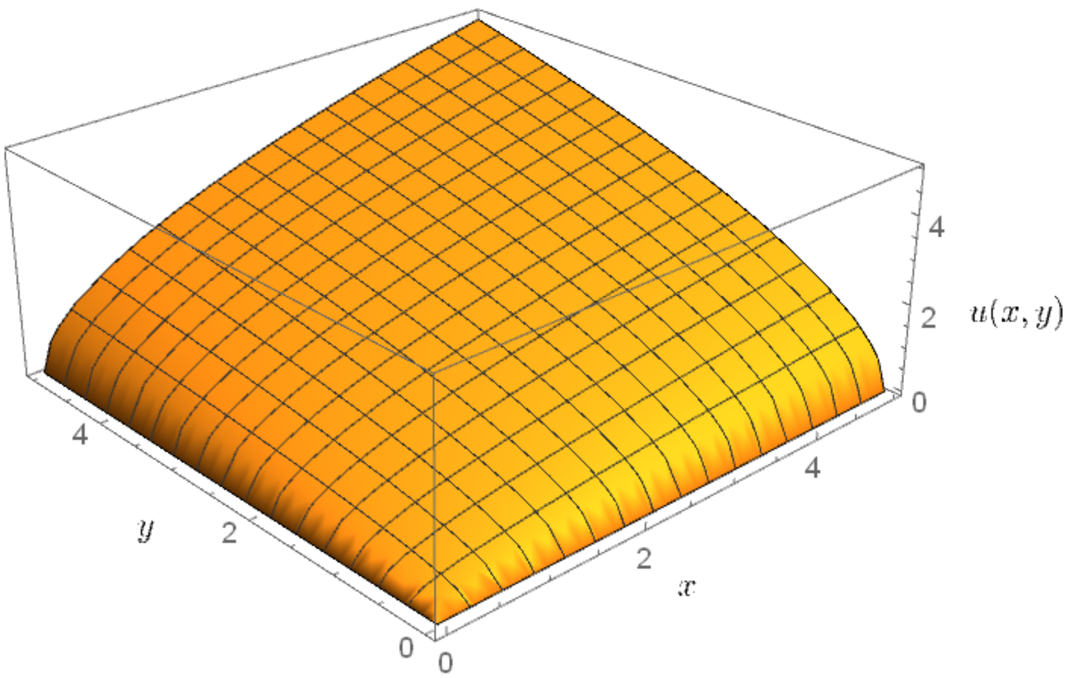
Consumer Theory: Utility Functions

<u>An Example:</u> $u(x,y) = 2x + y$



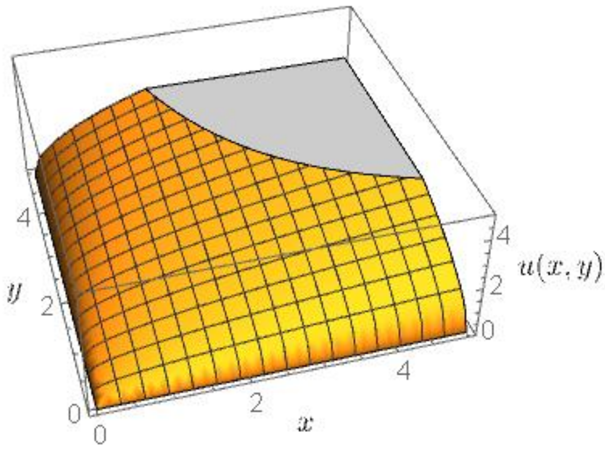
Consumer Theory: Utility Functions

<u>Another Example:</u> $u(x,y) = x^{1/2}\ y^{1/2}$



Consumer Theory: Utility Functions

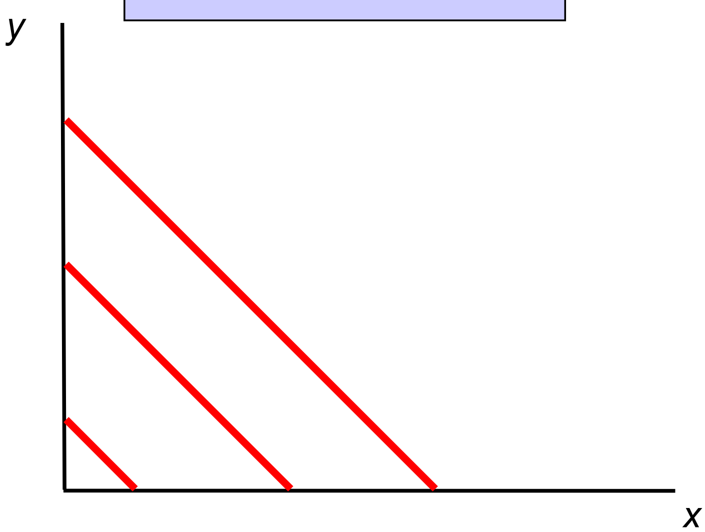
Like on a map, indifference curves are the contour lines of the "utility mountain":



Consumer Theory: Utility Functions

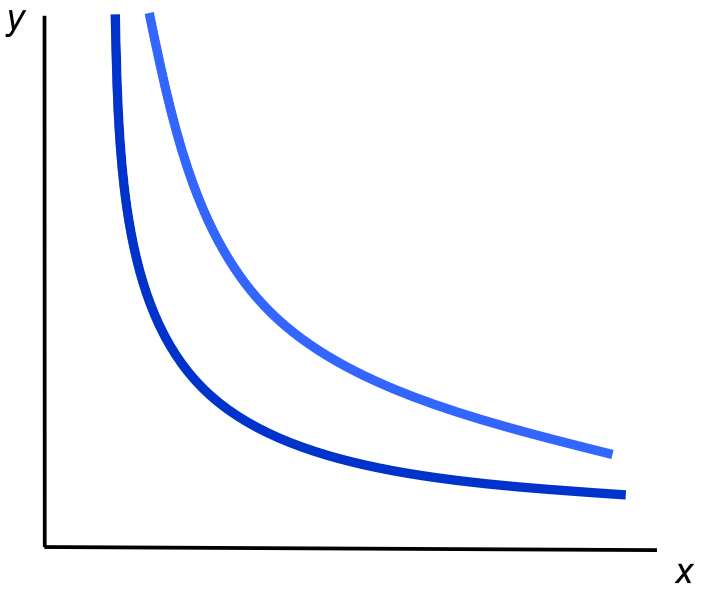
We can use utility functions to derive indifference curves:

$u(x,y)=ax+y,\ a>0$



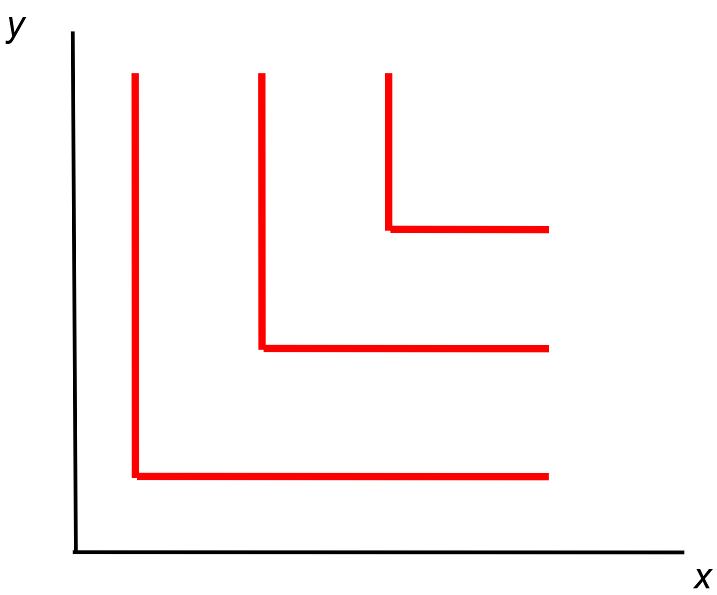
Consumer Theory: Utility Functions

$u(x,y)=x^a y^{1-a},\ 0 < a < 1$



Consumer Theory: Utility Functions

$u(x,y)=\min\{ax,y\},\ a>0$



Consumer Theory: Utility Functions

We have seen before that the numerical values that a utility assigns to the alternative bundles do not have an intrinsic meaning other than allowing us to order bundles according to the welfare they provide to the consumer:

If $u: \Re^2_+ \rightarrow \Re$ is a utility function and $g: \Re \rightarrow \Re$ is an increasing function, then the utility function $v: \Re^2_+ \rightarrow \Re$ given by $v(x,y)=g(u(x,y))$ represents ***the same preferences*** as u.

Thus, we may say that a utility function provides an ordinal (not cardinal) representation of a consumer's preferences.

Consumer Theory: Utility Functions

But under which conditions do we know that we can represent a consumer's preferences using a utility function?

Axioms **A1**, **A2** and **A4** imply the existence of a continuous utility function, $u: \Re^2_+ \rightarrow \Re$, that represents the consumer's preferences.

Axiom **A3** implies that the function $u(x,y)$ is non-decreasing in x and y, and is increasing in (x,y).

Axiom **A5** implies that u is (quasi-)concave.